Expression Templates

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Abstract: *Expression Templates* is a new C++ technique for passing expressions as function arguments. The expression can be inlined into the function body, which results in faster and more convenient code than C-style callback functions. This technique can also be used to evaluate vector and matrix expressions in a single pass without temporaries. In preliminary benchmark results, one compiler evaluates vector expressions at 95-99.5% efficiency of hand-coded C using this technique (for long vectors). The speed is 2-15 times that of a conventional C++ vector class.

Introduction

Passing an expression to a function is a common occurrence. In C, expressions are usually passed using a pointer to a callback function containing the expression. For example, the standard C library routines qsort(), lsearch(), and bsearch() accept an argument `int (*cmp)(void*, void*)` which points to a user-defined function to compare two elements. Another common example is passing mathematical expressions to functions. The problem with callback functions is that repeated calls generate a lot of overhead, especially if the expression which the function evaluates is simple. Callback functions also slow pipelined machines, since the function calls force the processor to stall.

The technique of expression templates allows expressions to be passed to functions as an argument and inlined into the function body. Here are some examples:

```
// Integrate a function from 0 to 10
DoublePlaceholder x;
double result = integrate(x/(1.0+x), 0.0, 10.0);

// Count the elements between 0 and 100 in a collection of int
List<int> myCollection;
IntPlaceholder y;
int n = myCollection.count(y >= 0 && y <= 100);

// Fill a vector with a sampled sine function
Vector<double> myVec(100);
Vector<double>::placeholderType i;
myVec.fill(sin(2*M_PI*i/100));  // eqn in terms of element #
```

In each of these examples, the compiler produces a function instance which contains the expression inline. The technique has a similar effect to Jensen's Device in ALGOL 60 [1].

Expression templates also solve an important problem in the design of vector and matrix class libraries. The technique allows developers to write code such as

```
Vector<double> y(100), a(100), b(100), c(100), d(100);
y = (a+b)/(c-d);
```

and have the compiler generate code to compute the result in a single pass, without using temporary vectors.

So how does it work?

The trick is that the expression is parsed at compile time, and stored as nested template arguments of an "expression type". Let's work through a simplified version of the first example, which passes the mathematical function

$$f(x) = \frac{x}{1+x}$$

to an integration routine. We'll invoke a function evaluate() which will evaluate f(x) at a range of points:

```
int main()
{
    DExpr<DExprIdentity> x;         // Placeholder
    evaluate(x/(1.0+x), 0.0, 10.0);
    return 0;
}
```

For this example, an expression is represented by an instance of the class DExpr<A> (short for *double expression*). This class is a wrapper which disguises more interesting expression types. DExpr<A> has a template parameter (A) representing the interesting expression, which can be of type DBinExprOp (a binary operation on two subexpressions), DExprIdentity (a placeholder for the variable ***x***), or DExprLiteral (a constant or literal appearing in the expression). For example, the simple subexpression "1.0+x" would be represented by the expression type shown in Figure 1. These expression types are inferred by the compiler at compile time.

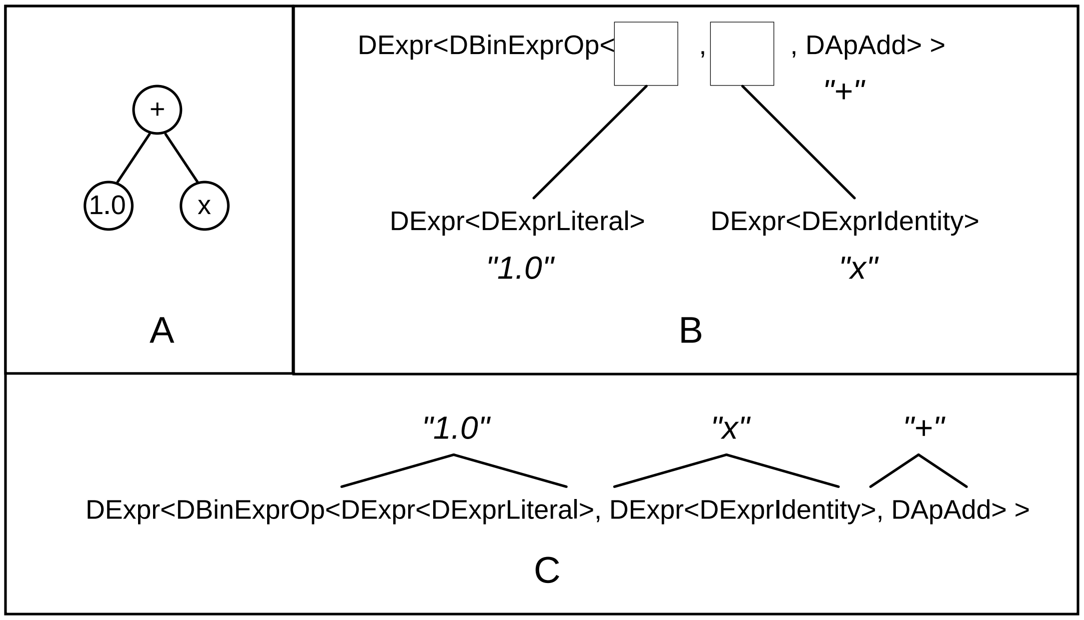


***Figure 1**. (**A**) Parse tree of "1.0+x", showing the similarity to the expression type generated by the compiler (**B**). The type DApAdd is an applicative template class which represents an addition operation. In (**C**), the type name is shown in a non-expanded view. For this implementation, type names are similar to a postfix traversal of the expression tree.*

To write a function which accepts an expression as an argument, we include a template parameter for the expression type. Here is the code for the evaluate() routine. The parenthesis operator of **expr** is overloaded to evaluate the expression:

```
template<class Expr>
void evaluate(DExpr<Expr> expr, double start, double end)
{
    const double step = 1.0;

    for (double i=start; i < end; i += step)
        cout << expr(i) << endl;
}
```

When the above example is compiled, an instance of evaluate() is generated which contains code equivalent to the line

```
cout << i/(1.0+i) << endl;
```

This happens because when the compiler encounters "x/(1.0+x)" in

```
evaluate(x/(1.0+x), 0.0, 10.0);
```

it uses the return types of overloaded + and / operators to infer the expression type shown in Figure 2. Note that although the expression *object* is not instantiated until run time, the expression *type* is inferred at compile time. An instance of this expression type is passed to the evaluate() function as the first argument. When the fragment **expr(i)** is encountered, the compiler builds the expression inline, substituting **i** for the placeholder **x**. The mechanics of this are explained further on.

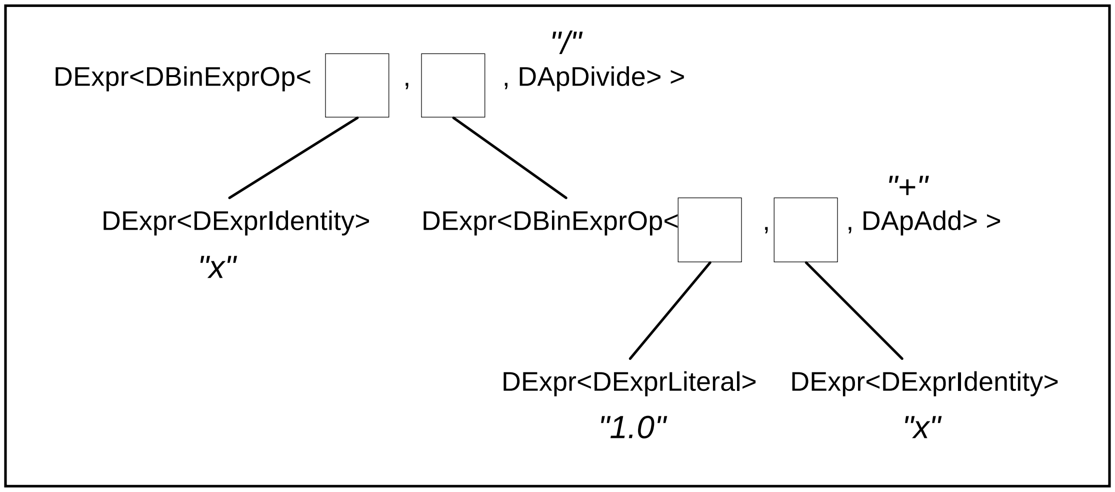


Figure 2. *Template instance generated by x/(1.0+x): DExpr<DBinExprOp< DExpr <DExprIdentity>, DExpr<DBinExprOp<DExpr<DExprLiteral>, DExpr<DExprIdentity>, DApAdd>>, DApDivide>>*

As an example of how the return types of operators cause the compiler to infer expression types, here is an operator/() which produces a type representing the division of two subexpressions DExpr<A> and DExpr<B>:

```
template<class A, class B>
DExpr<DBinExprOp<DExpr<A>, DExpr<B>, DApDivide> >
operator/(const DExpr<A>& a, const DExpr<B>& b)
{
    typedef DBinExprOp<DExpr<A>, DExpr<B>, DApDivide> ExprT;
    return DExpr<ExprT>(ExprT(a,b));
}
```

The return type contains a DBinExprOp, which represents a binary operation, with template parameters DExpr<A> and DExpr<B> (the two subexpressions being added), and DApDivide, which is an *applicative template class* encapsulating the division operation. The return type is a DBinExprOp<...> disguised by wrapping it with a DExpr<...> class. If we didn't disguise everything as DExpr<>, we would have to write eight different operators to handle the combinations of DBinExprOp<>, DExprIdentity, and DExprLiteral which can occur with operator/(). (More if we wanted unary operators!)

The typedef ExprT is the type DBinExprOp< ... , ... , DApDivide> which is to be wrapped by a DExpr<...>. Expression classes take constructor arguments of the embedded types; for example, DExpr<A> takes a const A& as a constructor argument. So DExpr<ExprT> takes a constructor argument of type const ExprT&. These arguments are needed so that instance data (such as literals which appear in the expression) are preserved.

The idea of applicative template classes has been borrowed from the Standard Template Library (STL) developed by Alexander Stepanov and Meng Lee, of Hewlett Packard Laboratories [2]. The STL has been accepted as part of the ISO/ANSI Standard C++ Library. In the STL, an applicative template class provides an inline operator() which applies an operation to its arguments and returns the result. For expression templates, the application function can (and should) be a static member function. Since operator() cannot be declared static, an apply() member function is used instead:

```
// DApDivide -- divide two doubles
class DApDivide {
public:
    static inline double apply(double a, double b)
    { return a/b; }
};
```

When a DBinExprOp's operator() is invoked, it uses the applicative template class to evaluate the expression inline:

```
template<class A, class B, class Op>
class DBinExprOp {
    A a_;
    B b_;

public:
    DBinExprOp(const A& a, const B& b)
      : a_(a), b_(b)
    { }

    double operator()(double x) const
    { return Op::apply(a_(x), b_(x)); }
};
```

It is also possible to incorporate functions such as exp() and log() into expressions, by defining appropriate functions and applicative templates. For example, an expression object representing a normal distribution can be easily constructed:

```
double mean=5.0, sigma=2.0;   // mathematical constants
DExpr<DExprIdentity> x;
evaluate( 1.0/(sqrt(2*M_PI)*sigma) * exp(sqr(x-mean)/
    (-2*sigma*sigma)), 0.0, 10.0);
```

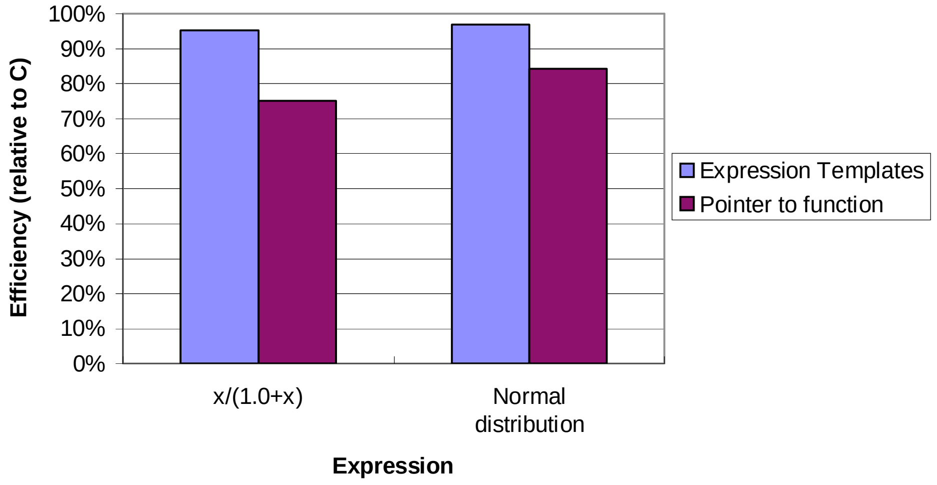
One of the neat things about expression templates is that mathematical constants are evaluated only once at run time, and stored as literals in the expression object. The evaluate() line above is equivalent to:

```
double __t1 = 1.0/(sqrt(2*M_PI)*sigma);
double __t2 = (-2*sigma*sigma);
evaluate( __t1 * exp(sqr(x-mean)/__t2), 0.0, 10.0);
```

Another advantage is that it's very easy to pass parameters (such as the mean and standard deviation) -- they're stored as variables inside the expression object. With pointers-to-functions, these would have to be stored as globals (messy), or passed as arguments in each call to the function (expensive).

Figure 3 shows a benchmark of a simple integration function, comparing callback functions and expression templates to manually inlined code. Expression template versions ran at 95% the efficiency of hand-coded C. With larger expressions, the advantage over pointers-to-functions shrinks. This advantage is also diminished if the function using the expression (in this example, integrate()) does significant computations other than evaluating the expression.

Figure 3. Efficiency for integrate() as compared to hand-coded C (1000 evaluations per call)



It is possible to generalize the classes presented here to expressions involving arbitrary types (instead of just doubles). Expression templates can also be used in situations where multiple variables are needed -- such as filling out a matrix according to an equation.

Since most compilers do not yet handle member function templates, a virtual function trick has to be used to pass expressions to member functions of a class. This technique is illustrated in the next example, which solves a significant outstanding problem in the design of matrix and vector class libraries.

Optimizing Vector Expressions

C++ class libraries are wonderful for scientific and engineering applications, since operator overloading makes it possible to write algebraic expressions containing vectors and matrices:

```
DoubleVec y(1000), a(1000), b(1000), c(1000), d(1000);
y = (a+b)/(c-d);
```

Until now, this level of abstraction has come at a high cost, since vector and matrix operators are usually implemented using temporary vectors [3]. Evaluating the above expression using a conventional class library generates code equivalent to:

```
DoubleVec __t1 = a+b;
DoubleVec __t2 = c-d;
DoubleVec __t3 = __t1/__t2;
y = __t3;
```

Each line in the code above is evaluated with a loop. Thus, four loops are needed to evaluate the expression y=(a+b)/(c-d). The overhead of allocating storage for the temporary vectors (__t1, __t2, __t3) can also be significant, particularly for small vectors. What we would like to do is evaluate the expression in a single pass, by fusing the four loops into one:

```
// double *yp, *ap, *bp, *cp, *dp all point into the vectors.
// double *yend is the end of the y vector.
do {
    *yp = (*ap + *bp)/(*cp - *dp);
    ++ap; ++bp; ++cp; ++dp;
} while (++yp != yend);
```

By combining the ideas of expression templates and iterators, it is possible to generate this code automatically, by building an expression object which contains vector iterators rather than placeholders.

Here is the declaration for a class DVec, which is a simple "vector of doubles". DVec declares a public type iterT, which is the iterator type used to traverse the vector (for this example, an iterator is a double*). DVec also declares Standard Template Library compliant begin() and end() methods to return iterators positioned at the beginning and end of the vector:

```
class DVec {

private:
    double* data_;
    int length_;

public:
    typedef double* iterT;

    DVec(int n)
        : length_(n)
    { data_ = new double[n]; }

    ~DVec()
    { delete [] data_; }

    iterT begin() const
    { return data_; }

    iterT end() const
    { return data_ + length_; }

    template<class A>
    DVec& operator=(DVExpr<A>);
};
```

Expressions involving DVec objects are of type DVExpr<A>. An instance of DVExpr<A> contains iterators which are positioned in the vectors being combined. DVExpr<A> lets *itself* be treated as an iterator by providing two methods: `double` `operator*()` evaluates the expression using the current elements of all iterators, and `operator++()` increments all the iterators:

```
template<class A>
class DVExpr {

private:
    A iter_;

public:
    DVExpr(const A& a)
        : iter_(a)
    { }

    double operator*() const
    { return *iter_; }

    void operator++()
    { ++iter_; }
};
```

The constructor for DVExpr<A> requires a const A& argument, which contains all the vector iterators and other instance data (eg. constants) for the expression. This data is stored in the iter_ data member. When a DVExpr<A> is assigned to a Dvec, the Dvec::operator=() method uses the overloaded * and ++ operators of DVExpr<A> to store the result of the vector operation:

```
template<class A>
DVec& DVec::operator=(DVExpr<A> result)
{
    // Get a beginning and end iterator for the
vector
    iterT iter = begin(), endIter = end();

    // Store the result in the vector
    do {
        *iter = *result;   // Inlined expression
        ++result;
    } while (++iter != endIter);

    return *this;
}
```

Binary operations on two subexpressions or iterators A and B are represented by a class DVBinExprOp<A,B,Op>. The operation itself (Op) is implemented by an applicative template class. The prefix ++ and unary * (dereferencing) operators of DVBinExprOp<A,B,Op> are overloaded to invoke the ++ and unary * operators of the A and B instances which it contains.

As before, operators build expression types using nested template arguments. Several versions of the each operator are required to handle the combinations in which DVec, DVExpr<A>, and numeric constants can appear in expressions.

For a simple example of how expressions are built, consider the code snippet

```
DVec a(10), b(10);
y = a+b;
```

Since both 'a' and 'b' are of type DVec, the appropriate operator+() is:

```
DVExpr<DVBinExprOp<DVec::iterT,DVec::iterT,DApAdd> >
operator+(const DVec& a, const DVec& b)
{
    typedef DVBinExprOp<DVec::iterT,DVec::iterT,DApAdd> ExprT;
    return DVExpr<ExprT>(ExprT(a.begin(),b.begin()));
}
```

All the operators which handle DVec types invoke DVec::begin() to get iterators. These iterators are bundled as part of the returned expression object.

As before, it's possible to add to this scheme so that literals can be used in expressions, as well as unary operators (such as x = -y), and functions (sin, exp). It's also not difficult to write a templatized version of DVec and the DVExpr classes, so that vectors of arbitrary types can be used. By using another template technique called "traits", it's even possible to implement standard type promotions for vector algebra, so that adding a Vec<int> to a Vec<double> produces a Vec<double>, etc.

Figure 4 shows the template instance inferred by the compiler for the expression y = (a+b)/(c-d). Although long and ugly type names result, the vector expression is evaluated entirely inline in a single pass by the assignResult() routine.

To evaluate the efficiency of this approach, benchmarks were run comparing its performance to hand-crafted C code, and that of a conventional vector class (ie. one which evaluates expressions using temporaries). The performance was measured for the expression y=a+b+c.

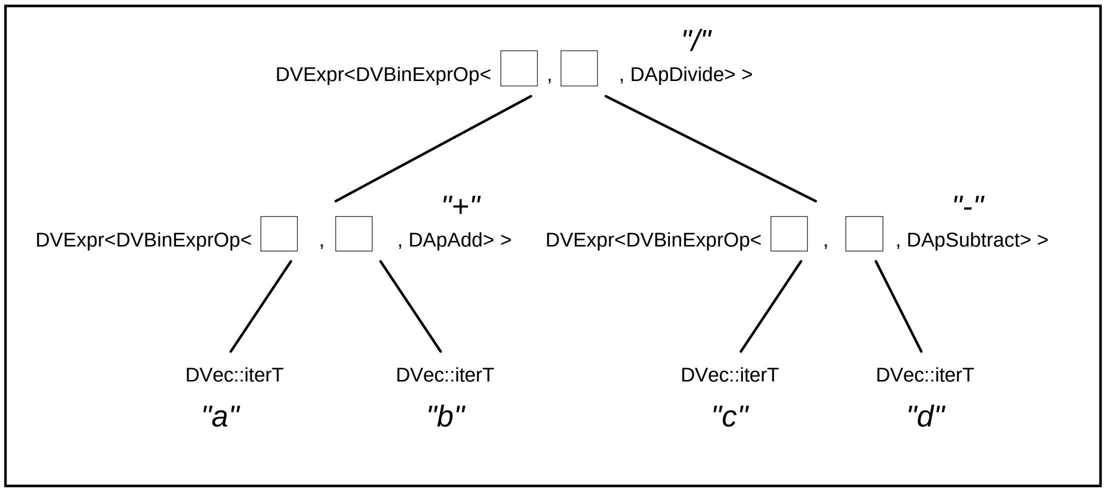

"/"
DVExpr<DVBinExprOp< , , DApDivide> >
"+"
DVExpr<DVBinExprOp< , , DApAdd> >
"-"
DVExpr<DVBinExprOp< , , DApSubtract> >
DVec::iterT
"a"
DVec::iterT
"b"
DVec::iterT
"c"
DVec::iterT
"d"

Figure 4. *Type name for the expression (a+b)/(c-d): DVExpr<DVBinExprOp<DVExpr< DVBinExprOp<DVec::iterT, DVec::iterT, DApAdd>>, DVExpr<DVBinExprOp< DVec::iterT, DVec::iterT, DApSubtract>>, DApDivide>>*

Figure 5 shows the performance of the expression template technique, as compared to hand-coded C, using the Borland C++ V4.0 compiler under MS-DOS. With a few adjustments of compiler options and tuning of the assignResult() routine, the same assembly code was generated by the expression template technique as hand coded C. The poorer performance for smaller vectors was due to the time spent in the expression object constructors. For longer vectors (eg., length 100) the benchmark results were 95-99.5% the speed of hand coded C.

Figure 5. Performance relative to hand-coded C

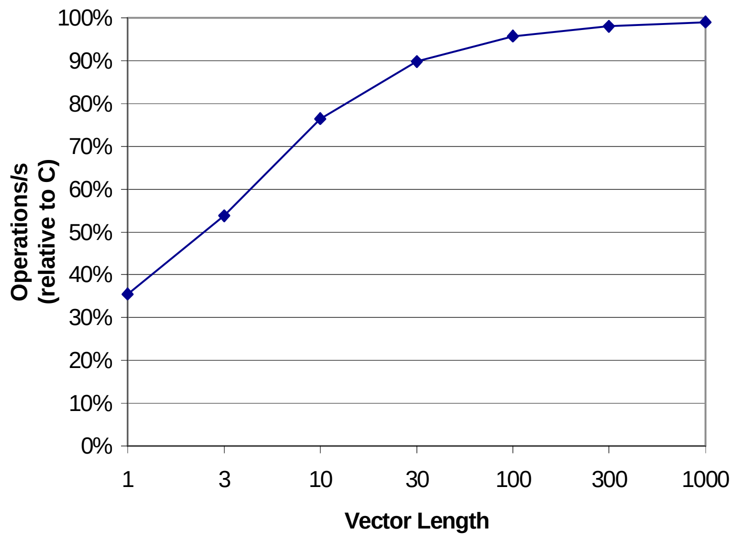
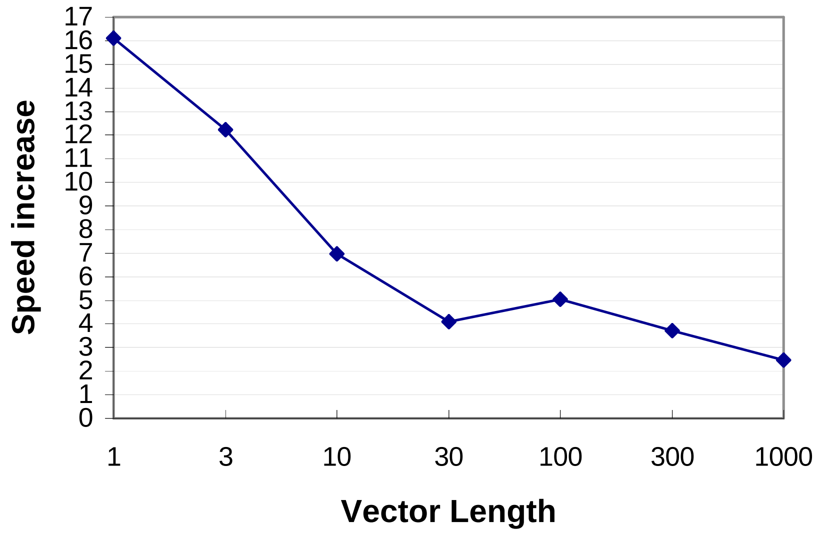


Figure 6 compares the expression template technique to a conventional vector class which evaluates expressions using temporaries. For short vectors (up to a length of 20 or so), the overhead of setting up temporary vectors caused the conventional vector class to have very poor performance, and the expression template technique was 8-12 times faster. For long vectors, the performance of expression templates settles out to twice that of the conventional vector class.

Figure 6. Speed increase over a conventional vector class



Conclusions

The technique of expression templates is a powerful and convenient alternative to C style callback functions. It allows logical and algebraic expressions to be passed to functions as arguments, and inlined directly into the function body. Expression templates also solve the problem of evaluating vector and matrix expressions in a single pass without temporaries.

References

[1] Rutihauser, H. “Description of ALGOL 60,” volume 1a of *Handbook for Automatic Computation*. Springer-Verlag, 1967.

[2] Stepanov, A. and Meng Lee, “The Standard Template Library”. ANSI X3J16-94-0095/ISO WG21-NO482.

[3] Budge, K. G. C++ Optimization and Excluding Middle-Level Code, *Proceedings of the Second Annual Object-Oriented Numerics Conference*, pp 107-121, Sunriver, Oregon, April 24-27, 1994.